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Interacting Entropy-Corrected New Agegraphic K-essence, Tachyon and Dilaton Scalar Field Models in Non-flat Universe

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Abstract We consider the new agegraphic dark energy model with the help of the quantum corrections to the entropy-area relation in the setup of loop quantum gravity. Employing this new form of dark energy so called entropy-corrected new agegraphic dark energy (ECNADE), we investigate the model of interacting dark energy and derive its equation of state (EoS). We study the correspondence between the K-essence, tachyon and dilaton scalar fields with the interacting (ECNADE)in the non-flat FRW universe. Moreover, we reconstruct the corresponding scalar potentials which describe the dynamics of the scalar field.

Keywords Tachyon · Dilaton · K-essence · Scalar fields · Dark energy

1 Introduction

One of the most outstanding developments in the last decade is the discovery that our universe is undergoing the phase of accelerating expansion [1–5]. “Dark energy”, an unknown exotic vacuum energy responsible for propelling the universe, is one of the deepest mysteries in cosmology. The dark energy possesses negative pressure $p < 0$ and positive energy density $\rho > 0$ which is related by the equation of state $p = \omega\rho$. Astrophysical data shows that about two-third of the critical energy is stored in the dark energy component apart from dark matter which contains only one third of the critical energy density. One possible source of this cosmic expansion can be explained by the general theory of relativity with the cosmological constant Λ. Although the cosmological constant is the most obvious choice, but it suffers from coincidence problem and the fine-tuning problems [6]. To overcome these problems, several alternative models have been suggested; among them are dynamical scalar field ϕ with suitably defined scalar field potential $V(\phi)$ termed as quintessence [7–10],

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quintom [11–13], k-essence [14–16], tachyon [17–20], phantom [21–23], dilatonic ghost condensate [24–26], to name a few. In addition, there are other proposals on dark energy such as interacting dark energy models [27–31], braneworld model [32, 33] and Chaplygin gas models [34, 35] etc.

In the last few years, the holographic dark energy (HDE) models [36, 37] and agegraphic dark energy (ADE) models [38] have received a considerable interest. Holographic principle is a speculative conjecture about proposed quantum theories of gravities . According to the holographic principle, the information contained in a volume may be described by a theory that lies on the boundary of that space [39]. In the study of thermodynamics of the black hole, there is a maximum entropy in a box of size L, known as the Bekenstein Hawking entropy bound $S \backsim m_p^2 L^2$ which scales as the area of the box $A \backsim L^2$ and $m_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass. To avoid the breakdown of quantum field theory in the frame work of quantum gravity, Cohen et al [40] proposed that the entropy for an effective theory should satisfy $L^3\Lambda^3 \le S^{3/4} \le (m_p^2 L)^{3/2}$. Here L is associated with the size of a region which gives an infra-red cut-off while Λ corresponds to the ultra-violet cut-off. The last expression can be transformed to $\rho_\Lambda = 3n^2 m_p^2 L^{-2}$, where $3n^2$ is for convenience. In Einstein theory of gravity, the definition of holographic dark energy requires the entropy-area relationship $S \backsim A \backsim L^2$, where A is the area of the horizon. However in the context of loop quantum gravity (LQG), this relationship gets modified from the inclusion of quantum effects. The quantum corrections provided to the entropy-area relationship adds up extra terms in Einstein Hilbert action and vise versa [41, 42]. The corrected entropy is

$$S = \frac{A}{4G} + \tilde{\gamma} \ln\left(\frac{A}{4G}\right) + \tilde{\beta}, \tag{1}$$

where $\tilde{\gamma}$ and $\tilde{\beta}$ are constants of order unity. The exact values of these parameters are unknown and still a matter of debate. These corrections usually come into view in the black hole entropy in LQG due to thermal and quantum fluctuations [43–46]. The entropy-corrected holographic dark energy (ECHDE) is given by [47, 48]

$$\rho_\Lambda = 3n^2 m_p^2 L^{-2} + \gamma L^{-4} \ln(m_p^2 L^2) + \beta L^{-4}, \tag{2}$$

where γ and β are dimensionless constants of order unity. Clearly if we put $\gamma = \beta = 0$, we arrive at the holographic dark energy (HDE) model. In the energy density of the ECHDE (2), the last two terms can be comparable to the first term only when L is very small, so the corrections make sense only at the early stage of the universe. Later on when the universe expands and become large, the ECHDE behaves as HDE.

Though HDE is considered to be an interesting candidate for dark energy, but there are some drawbacks that appear in HDE model. For instance, by choosing the future event horizon of the universe as the length scale, though the HDE model does not seem to contradict the observed value of dark energy in the universe and can propel the universe to an accelerated expansion phase. However, the current properties of the dark energy determined by the future evolution of the universe might violate causality. Moreover, it has been argued that the HDE model might be in contradiction to the age of the universe [49]. The ECHDE model also has a problem that it is suitable for earlier epoch of the universe while it behaves as HDE as the universe gets large.

To alleviate the causality and age problems, recently Cai [38] has proposed a model, dubbed “agegraphic dark energy” (ADE) based on Karolyhazy uncertainty relation $\delta t = \lambda t_p^{2/3} \tau^{1/3}$ [50–52], λ is a numerical factor of order one and t_p is the Planck time. Following the Karolyhazy relation, Maziashvili argued that the energy density of metric fluctuation of Minkowski spacetime can be written as $\rho_\Lambda \backsim \frac{1}{t_p^2\tau^2} \backsim \frac{m_p^2}{\tau^2}$, where the time scale τ is chosen to be the age of the universe $T = \int_0^a \frac{d\tilde{a}}{H\tilde{a}}$ and the energy density of the agegraphic dark energy can be expressed as $\rho_\Lambda = 3n^2 m_p^2 T^{-2}$ [38]. Since in the ADE model the age of the universe is taken as the length measure instead of the horizon distance, so the causality problem that appears in the HDE model can be avoided. However, the ADE model might contain an inconsistency [53]. So soon after the original ADE model, the authors [54] proposed an alternative model of dark energy so called the " new agegraphic dark energy" (NADE) and its energy density is defined by

$$\rho_\Lambda = 3n^2 m_p^2 \eta^{-2}, \tag{3}$$

where η is conformal time of the FRW-universe and is written as

$$\eta = \int \frac{dt}{a} = \int_0^a \frac{da}{Ha^2}. \tag{4}$$

In this paper, our aim is to extend the study of NADE by introducing the quantum corrections to the entropy-area relation. We investigate the entropy-corrected version of the interacting NADE model so called the entropy-corrected new agegraphic dark energy (ECNADE) and see its effects in the non-flat universe. In the setup of LQG, the ECNADE model can be obtained by taking the length L in (2) to be the conformal time η given by (4). Hence under this assumption, the energy density of the ECNADE can be written as [47, 48]

$$\rho_\Lambda = 3n^2 m_p^2 \eta^{-2} + \gamma \eta^{-4} \ln(m_p^2 \eta^2) + \beta \eta^{-4}, \tag{5}$$

where γ and β are dimensionless constants of order one. Notice that if we suppose γ and β to be zero in (5), we turn up the NADE model (3).

The plan of the present work is as follows. In Sect. 2, we derive the equation of state parameter for the interacting ECNADE model in a non-flat Friedmann-Robertson-Walker (FRW) universe. In Sect. 3, we propose a correspondence between the interacting ECNADE and the K-essence, tachyon and dilaton scalar fields. In each case, we reconstruct the potential and dynamics for these scalar fields which depict the accelerated expansion. Finally we present the conclusion in Sect. 4.

2 Interacting ECNADE Model

We assume the background to be a spatially homogeneous and isotropic FRW spacetime, given by

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]. \tag{6}$$

Here $a(t)$ is the dimensionless scale factor which is an arbitrary function of time and k is represent the curvature parameter which has dimensions of *length*$^{-2}$. For the values $k = -1, 0, 1$, the above metric represents the spatially open, flat and closed FRW universe respectively. The first Friedmann equation for the non-flat FRW-spacetime containing the dark energy and dark matter is

$$H^2 + \frac{k}{a^2} = \frac{1}{3m_p^2}(\rho_\Lambda + \rho_m). \tag{7}$$

Here $H = \dot{a}/a$ is the Hubble constant while ρ_Λ and ρ_m represent the energy densities of dark energy and matter respectively. Let us define the dimensionless energy density parameters as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3m_p^2 H^2}, \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{\rho_\Lambda}{3m_p^2 H^2}, \qquad \Omega_k = \frac{k}{(aH)^2}. \tag{8}$$

With the help of these parameters, the Friedmann (7) takes the form

$$1 + \Omega_k = \Omega_\Lambda + \Omega_m. \tag{9}$$

Employing the relationship $\Omega_\Lambda = \frac{\rho_\Lambda}{3H^2 m_p^2}$, we get

$$\eta^2 = \frac{3n^2 m_p^2 + \gamma\eta^{-2}\ln(m_p^2\eta^2) + \beta\eta^{-2}}{3m_p^2 H^2 \Omega_\Lambda}. \tag{10}$$

Let us assume an interaction $Q = \Gamma\rho_\Lambda$ between ECNADE and the cold dark matter (CDM) having $\omega_m = 0$. The resulting energy conservation equations for ECNADE and CDM are

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = -Q, \tag{11}$$

$$\dot{\rho}_m + 3H\rho_m = Q. \tag{12}$$

Here overdot represents the differentiation with respect to cosmic time t. These interacting models describe an energy flow between the dark energy and dark matter so that no component shows energy conservation independently. If two species are present in dominant form, they are definitely supposed to interact. We choose $\Gamma = 3b^2 H(\frac{1+\Omega_k}{\Omega_\Lambda})$ as the decay rate of the ECNADE component into CDM with a coupling constant b^2. The importance of interacting model also emerges as it is good fit to the expansion history of the universe as determined by the Supernovae and cosmic microwave background [1–5].

Differentiate (5) with respect to time and using $\dot{\eta} = \frac{1}{a}$ and (10), we have

$$\dot{\rho}_\Lambda = \frac{2\chi H m_p}{a}\sqrt{\frac{3\Omega_\Lambda}{3n^2 m_p^2 + \gamma\eta^{-2}\ln(m_p^2\eta^2) + \beta\eta^{-2}}}, \tag{13}$$

where $\chi \equiv \gamma\eta^{-4} - 2\gamma\eta^{-4}\ln(m_p^2\eta^2) - 2\beta\eta^{-4} - 3n^2 m_p^2\eta^{-2}$. Making use of the above (13) in (11), we obtain

$$\omega_\Lambda = -1 - \frac{2\chi}{3a}\left(\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2 m_p^2\eta^{-2} + \gamma\eta^{-4}\ln(m_p^2\eta^2) + \beta\eta^{-4}}\right) - b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right). \tag{14}$$

The above equation (14) represents the equation of state for the ECNADE model interacting with matter in a non-flat universe.

3 Correspondence Between ECNADE and K-essence, Tachyon and Dilaton Scalar Fields

The cosmological constant corresponds to a fluid with a constant equation of state $\omega = -1$. Now, the observations which put restriction on the value of ω to be close to that of cosmological constant, explain bit less about time evolution of ω. So we need to consider a model in which the EoS of dark energy evolves with time such as in inflationary cosmology. Scalar field models arise in string theory and are studied as promising choices for dark energy. So far, ample literature dealing with scalar field dark energy models is available [55–67]. It includes quintessence, K-essence, phantoms, tachyon, and dilaton among many. In this section, we investigate the correspondence between the interacting ECNADE model with the K-essence, tachyon and dilaton scalar fields in the non-flat FRW universe. To illustrate this correspondence, we first equate the interacting ECNADE density with the corresponding scalar field density. Then we compare the equation of state of the scalar field model with the EoS of the ECNADE.

3.1 Entropy-Corrected New Agegraphic K-essence Model

The idea of the K-essence scalar field was motivated from the Born-Infeld action of string theory [68, 69] and used as a source to explain the mechanism for producing the late time acceleration of the universe. The K-essence model is expressed by a general scalar field action which is function of ϕ and $X = \dot{\phi}^2/2$ and is given by [14–16]

$$S = \int d^4x\sqrt{-g}\; p(\phi, X), \tag{15}$$

where the Lagrangian density $p(\phi, X)$ corresponds to a pressure density as

$$p(\phi, X) = f(\phi)(-X + X^2), \tag{16}$$

and the energy density of the field ϕ as

$$\rho(\phi, X) = f(\phi)(-X + 3X^2). \tag{17}$$

The EoS parameter for the K-essence scalar field is obtained in the following way

$$\omega_K = \frac{p(\phi, X)}{\rho(\phi, X)} = \frac{X-1}{3X-1}. \tag{18}$$

After equating (18) with the ECNADE equation of state parameter (14), we determine the expression for X in the form

$$X = \frac{2 + \frac{2\chi}{3a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}} + b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}{4 + \frac{2\chi}{a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}} + 3b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}. \tag{19}$$

Using the above (19) and the relation $\dot{\phi}^2 = 2X$, the evolutionary form of the K-essence scalar field is determined to be

$$\dot{\phi} = \left(\frac{4 + \frac{4\chi}{3a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}} + 2b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}{4 + \frac{2\chi}{a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}} + 3b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}\right)^{1/2}, \tag{20}$$

which takes the form

$$\phi(a)-\phi(a_0)=\int_{a_0}^{a}\frac{1}{aH}\left(\frac{4+\frac{4\chi}{3a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}}+2b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}{4+\frac{2\chi}{a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}}+3b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}\right)^{1/2}da. \quad (21)$$

Here a_0 denotes the present value of the scale factor.

3.2 Entropy-Corrected New Agegraphic Tachyon Model

In recent years, a huge interest has been devoted in studying the inflationary model with the help of tachyon field. The tachyon field associated with unstable D-branes might be responsible for cosmological inflation in the early evolution of the universe, due to tachyon condensation near the top of the effective scalar potential [68, 69], which could suggests some new form of dark matter at late epoch [70]. A rolling tachyon has an interesting equation of state whose parameter smoothly interpolates between -1 and 0. This leads us to construct viable cosmological models by taking the tachyon as an appropriate candidate to explain inflation at high energy [71–74]. The effective Lagrangian density of tachyon matter is given by [17–20]

$$L=-V(\phi)\sqrt{1+\partial_\mu\phi\partial^\mu\phi}, \quad (22)$$

where $V(\phi)$ is the tachyon potential. The energy density and pressure for the tachyon are

$$\rho_T=\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \quad (23)$$

$$p_T=-V(\phi)\sqrt{1-\dot{\phi}^2}, \quad (24)$$

where $V(\phi)$ represents the tachyon potential. While the equation of state of the tachyon is given by

$$\omega_T=\frac{p_T}{\rho_T}=\dot{\phi}^2-1. \quad (25)$$

In order to develop the correspondence between the ECNADE and tachyon dark energy, we compare (25) and (14), and obtain

$$\dot{\phi}^2=-\frac{2\chi}{3a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{(3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4})}-b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right). \quad (26)$$

Now equating the (23) and (5), we get the following expression of potential energy for the tachyon

$$V(\phi)=(3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4})$$
$$\times\left(1+\frac{2\chi}{3a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{(3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4})}+b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\right)^{1/2}. \quad (27)$$

Equation (26) gives the folowing evolutionary form of the tachyon scalar field

$$\phi(a)-\phi(a_0)=\int_{a_0}^{a}\frac{1}{aH}\left(-\frac{\frac{2\chi}{3a}\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}}-b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\right)^{1/2}da. \tag{28}$$

3.3 Entropy-Corrected New Agegraphic Dilaton Model

A dilaton scalar field which exhibits the features of dark energy is usually originated from the lower-energy limit of string theory. This model is explained by a general four-dimensional effective low-energy string action. It has been shown [24–26] that a scalar field possessing negative kinetic term (usually known as phantom type scalar field) does not necessarily lead to inconsistencies provided that one takes an appropriate structure of higher order kinetic terms in the effective theory. The pressure density and the energy density of the dilaton dark energy model is given by [24–26]

$$p_D=-X+ce^{\lambda\phi}X^2, \tag{29}$$

$$\rho_D=-X+3ce^{\lambda\phi}X^2, \tag{30}$$

where c and λ are positive constants and $\dot{\phi}^2=2X$. The EoS parameter for the dilaton scalar field is given by

$$\omega_D=\frac{p_D}{\rho_D}=\frac{-1+ce^{\lambda\phi}X}{-1+3ce^{\lambda\phi}X}. \tag{31}$$

Following the same steps as done for the above cases, the comparison of (31) and (14) yields

$$ce^{\lambda\phi}X=\frac{2+\frac{2\chi}{3a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}}+b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}{4+\frac{2\chi}{a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}}+3b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}. \tag{32}$$

Insert the value of X i.e. $X=\dot{\phi}^2/2$ in the above equation (32), we get

$$ce^{\lambda\phi}\dot{\phi}^2=\frac{4+\frac{4\chi}{3a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}}+2b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}{4+\frac{2\chi}{a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}}+3b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}. \tag{33}$$

The above equation can be written as

$$e^{\frac{\lambda\phi}{2}}\dot{\phi}=\left(\frac{1}{c}\frac{4+\frac{4\chi}{3a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}}+2b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}{4+\frac{2\chi}{a}\frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}}+3b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}\right)^{1/2}, \tag{34}$$

and its integration yields

$$\phi(a) = \frac{2}{\lambda} \ln\Bigg[e^{\frac{\lambda\phi(a_0)}{2}} \\ + \frac{\lambda}{2\sqrt{c}} \int_{a_0}^{a} \frac{1}{aH} \left(\frac{4 + \frac{4\chi}{3a} \frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}} + 2b^2(\frac{1+\Omega_k}{\Omega_\Lambda})}{4 + \frac{2\chi}{a} \frac{\sqrt{\frac{3m_p^2\Omega_\Lambda}{3n^2m_p^2+\gamma\eta^{-2}\ln(m_p^2\eta^2)+\beta\eta^{-2}}}}{3n^2m_p^2\eta^{-2}+\gamma\eta^{-4}\ln(m_p^2\eta^2)+\beta\eta^{-4}} + 3b^2(\frac{1+\Omega_k}{\Omega_\Lambda})} \right)^{1/2} da \Bigg]. \quad (35)$$

4 Conclusion

In this paper we have discussed the new agegraphic dark energy model from the inclusion of quantum correction in the entropy-area relation so called ECNADE. This ECNADE interacts with cold dark matter in the non-flat FRW spacetime. Observational evidences implied that our universe is not perfectly flat and it possesses a small positive curvature ($\Omega_k = 0.02$) [75–77]. It is usually believed that the early inflation era leads to a flat universe, a contribution to the Friedmann equation from spatial curvature is still achievable if the number of e-folding is not very large [78]. Since the NADE model in the framework of quantum gravity is being used as a source to probe the nature of dark energy, so we have used the logarithmic corrected version of NADE which is usually motivated from the important feature of LQG. Employing this new definition of ECNADE, we have derived the equation of state parameter in the non-flat universe. We have studied a correspondence between the interacting ECNADE with the, K-essence, tachyon and dilaton scalar field models in order to see how these various candidates of dark energy are related to each other. Since the scalar field models are effective theories of an underlying theory of dark energy, so one can use these models according to the evolutionary behavior of the interacting ECNADE. For this we have reconstructed the potentials and the dynamics of these scalar field models which depicts accelerated expansion of the universe, according to the evolutionary behavior of the interacting ECNADE model.

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